

Simple Chosen-Ciphertext Security from Low-Noise LPN

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#### Outline



2 Low-Noise LPN

3 Applying Techniques of LWE-Based Schemes

4 Simple TBE Based on Low-Noise LPN



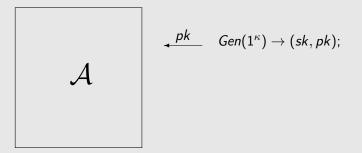


# A public key encryption PKE = (Gen, Enc, Dec) is called IND-CCA secure, if for every PPT adversary A holds:

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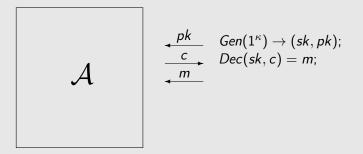


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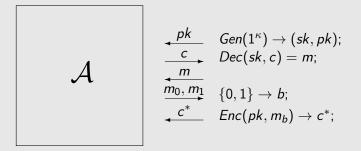


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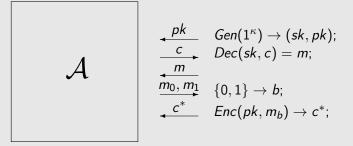


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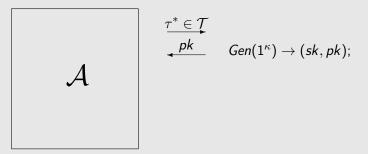
it is hard for  $\mathcal{A}$  to guess b.



A tag-based encryption TBE = (Gen, Enc, Dec) with a tag space  $\mathcal{T} = \{0, 1\}^{\kappa}$  is called weakly secure, if for every PPT adversary  $\mathcal{A}$  holds:

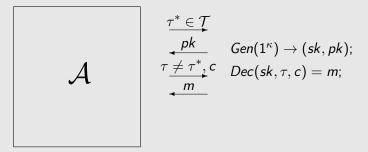


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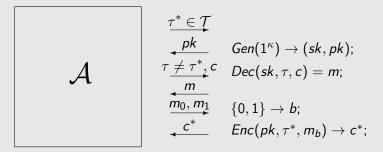


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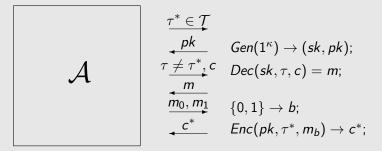


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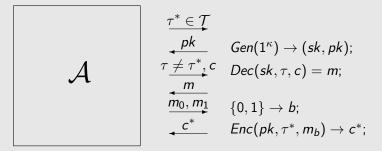
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#### Tag-Based Encryption (TBE)





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► A TBE is easier to construct than an IND-CCA PKE.

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#### Why Tag-Based Encryption?

- ► A TBE is easier to construct than an IND-CCA PKE.
- ► There are generic transformations from a weakly secure TBE to a IND-CCA PKE [BK05, Kil06, BCHK07].

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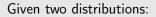
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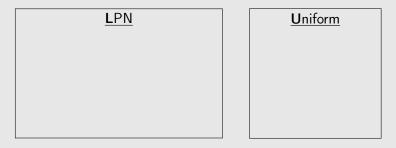


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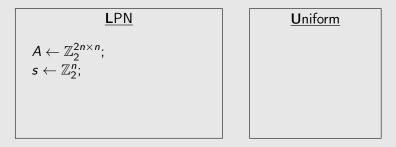


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LPN	<u>Uniform</u>
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A IND-CPA secure PKE by Alekhnovich [Ale03]. A TBE by Döttling et al. [DMQN12].

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#### TBE by Döttling et al. [DMQN12]

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- An encryption uses a  $B_{\tau}$  which is derived from  $B_1, \ldots, B_q$ .
- This results in a large public key ( $q \approx 400$ ).

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LWE-Based Trapdoor Mechanism [MP12]

► 
$$sk = T \in \{0,1\}^{\omega(n) \times \omega(n)}$$
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 is as close to uniform as  $B$ .



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- $B' := TA G\tau^*$  is as close to uniform as B.

• For 
$$c'_1 :\approx B's + G\tau s$$
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$$c_1' - Tc pprox G( au - au^*)s$$

and s is reconstructable for all  $\tau \neq \tau^*$ .

# Lattice-Based Trapdoor Mechanism



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and s is reconstructable for all  $\tau \neq \tau^*$ .

• For  $\tau = \tau^*$  some instances remain hard.



• 
$$A, B \in \mathbb{Z}_2^{2n \times n}, \ \mathcal{T} = \mathbb{F}_{2^n} \subset \mathbb{Z}_2^{n \times n}.$$



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- ► Either the noise is too big or *B* is not close to uniform.



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- Sample T such that the noise in c − Tc<sub>1</sub> is small, while B := TA is close to uniform.
- Either the noise is too big or *B* is not close to uniform.
- ► This approach does not immediately apply to LPN.

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## Replacing the Leftover Hash Lemma





## Using a Trapdoor with Low Entropy

• Sample T from  $\mathcal{B}_p^{2n \times 2n}$ ,  $p \in \Theta(1/\sqrt{n})$ .

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# Replacing the Leftover Hash Lemma



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- How to answer decryption queries?





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## The Two Trapdoors Approach

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## The Two Trapdoors Approach

- We use two trapdoors  $sk_0 = T_0$  and  $sk_1 = T_1$ .
- $pk = (A, B_0, B_1)$
- $sk_0$  is a trapdoor for  $(A, B_0)$  and  $sk_1$  for  $(A, B_1)$ .



### Switching the Public Key

*pk* = (A, B<sub>0</sub>, B<sub>1</sub>) is computationally indistinguishable from *pk'* = (A, B'<sub>0</sub>, B'<sub>1</sub>).



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- $sk_0$  and  $sk_1$  are now trapdoors for all  $\tau \neq \tau^* \in \mathcal{T}$ .
- ► Once pk' is used, decrypting ciphertexts with tag \(\tau^\*\) is hard, given sk\_0 and sk\_1.

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## A TBE Based on Low-Noise LPN



### The Construction

• Gen(1<sup>k</sup>): Output  $sk := T_0$ ,  $pk := (A, B_0 := T_0A, B_1 := T_1A, C)$ for  $T_0, T_1 \leftarrow \mathcal{B}_p^{2n \times 2n}$  and  $A, C \leftarrow \mathbb{Z}_2^{2n \times n}$ .

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- $Enc(pk, \tau, m)$ : Sample randomness  $s \leftarrow \mathbb{Z}_2^n$ . Output

$$c :\approx As,$$
  $c_0 :\approx (B_0 + G\tau)s,$   
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►  $Dec(sk, \tau, (c, c_0, c_1, c_2))$ : Reconstruct *s* from  $c_0 - T_0c \approx G\tau s$ . Check consistency of  $c_1$  with *s*. Reconstruct *m* from  $c_2 - Cs \approx Gm$ . Output *m*. Summary



#### Summary

- We construct a TBE, which can be transformed to an IND-CCA PKE.
- ► The security is based on the Low-Noise LPN assumption.
- pk is computationally indistinguishable from pk'.
- For pk', decrypting ciphertexts associated with  $\tau^*$  is hard.
- ► While switching *pk* to *pk'* two trapdoors are used to answer decryption queries.



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Many thanks for your attention!

#### **QUESTIONS?**

hgi Horst Görtz Institut für IT-Sicherheit 18/19

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